

The fields obtained from (2) and (3) are the so-called "transverse" electric fields since  $E_z(m, n)$  vanishes.

Following an identical procedure, the  $\bar{N}$  solution leads to

$$\begin{aligned} E'_{m,n} = \bar{N}_{m,n} = -\frac{j}{\gamma_0} \nabla \times \nabla \times \left\{ \hat{a}_z [K_0 J_m(\beta r) \right. \\ \left. + K_1 Y_m(\beta r)] e^{\pm jm\theta} \cos\left(\frac{n\pi}{b} z\right) \right\} \\ m, n = 0, 1, 2, \dots \quad (4) \end{aligned}$$

Again the corresponding magnetic field,  $\bar{H}'_{m,n}$ , can be found using (3). These fields are the so-called "transverse" magnetic fields since  $H'_{z(m,n)} = 0$ .

It is also possible to derive the magnetic fields from the  $\bar{M}$  and  $\bar{N}$  solutions by requiring that  $\hat{n} \cdot \bar{M} = \hat{n} \cdot \bar{N} = 0$  at  $z=0$  and  $z=b$ , and then use (3) to determine the components of the electric fields.

JOSEPH F. DIENST  
Microwave Electronics Lab.  
College of Engrg.  
Rutgers, the State University  
New Brunswick, N. J.

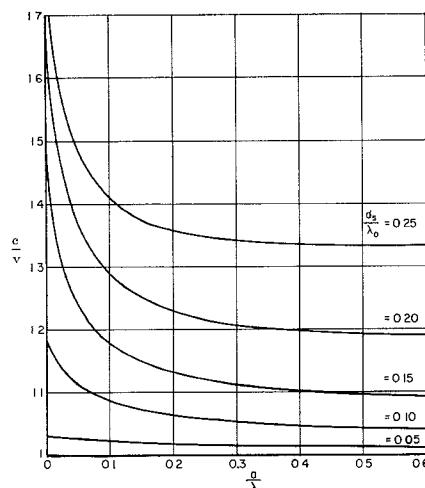


Fig. 1—Phase velocity ratio vs core thickness for the  $TM_0$  mode on an air-core sandwich. (Data also apply to a single slab over a ground plane, with an air space.)

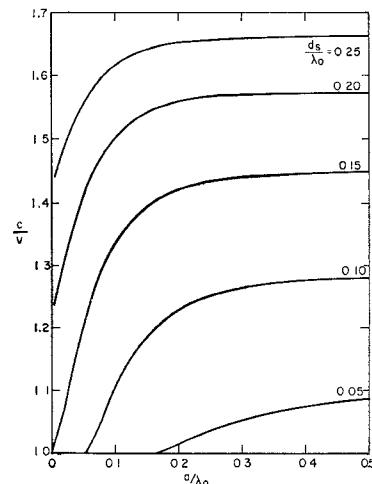


Fig. 2—Phase velocity ratio vs core thickness for the  $TE_1$  mode on an air-core sandwich. (Data also apply to a single slab over a ground plane, with an air space.)

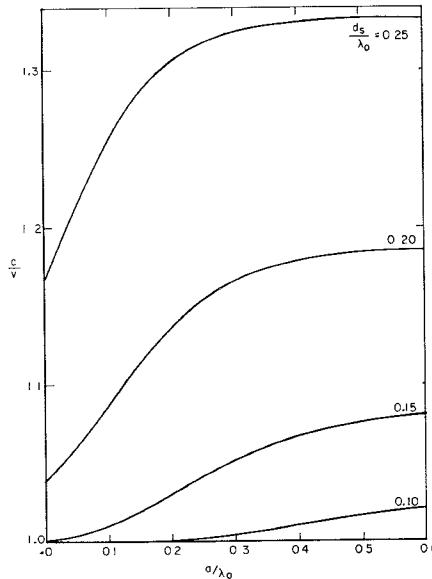


Fig. 3—Phase velocity ratio vs core thickness for the  $TM_1$  mode on an air-core sandwich.

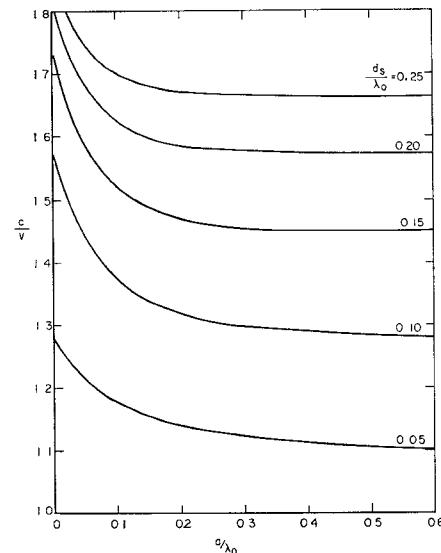


Fig. 4—Phase velocity ratio vs core thickness for the  $TE_0$  mode on an air-core sandwich.

### Surface Waves on Symmetrical Three-Layer Sandwiches\*

The theory of surface waves on plane dielectric slabs has been presented by Plummer and Hansen.<sup>1</sup> Additional numerical results are shown in Figs. 1 and 2 for the lowest order TM and TE modes that can exist on a grounded dielectric slab. The slab has thickness  $d_s$ , and a relative dielectric constant of 4. It is separated by an air gap of thickness  $a$  from the ground plane.  $c/v$  represents the ratio of the velocity of light in free space and the phase velocity of the surface wave. By image theory, these modes ( $TM_0$  and  $TE_1$ ) can also exist on a symmetrical, three-layer, air-core sandwich to which the given numerical data also apply.

Figs. 3 and 4 show similar data for the  $TM_1$  and  $TE_0$  modes. These modes disappear if a ground plane is inserted at the center of the sandwich. For this reason, these modes are usually ignored in the literature.

The fields of a surface wave decay as  $e^{-\alpha z}$ , with distance from the surface of the plane structure. The attenuation constant  $\alpha$ , is not independent but is directly related to the phase velocity by  $(\alpha \lambda_0)^2 = 4\pi^2 [(c/v)^2 - 1]$ , as shown in Fig. 5. This may be called a universal curve of  $\alpha \lambda_0$  vs  $c/v$ , because it applies to TE and TM modes on any lossless plane structure.

The results mentioned were obtained under a project sponsored by the Air Research and Development Command, USAF.

J. H. RICHMOND  
Antenna Lab.  
The Ohio State University  
Columbus, Ohio

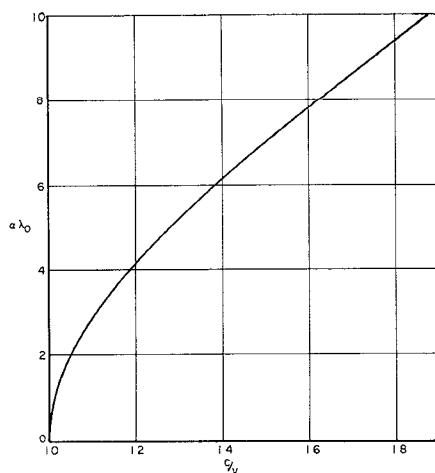


Fig. 5—Universal curve of  $\alpha \lambda_0$  vs  $c/v$ .

\* Received by the PGMTT, June 20, 1960.  
<sup>1</sup> R. Plummer and R. Hansen, *Proc. IEE*, pt. C, mono. 238R; May, 1957.